How to Build Sustainable Productivity Indexes

presented by

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1. Overview

- According to the OECD (2001, p.11) "productivity is commonly defined as a ratio of a volume measure of output to a volume measure of input use . . . there is no disagreement on this general notion".

- Some firms produce both good and bad outputs. We want to measure productivity in such a way that, if inputs and good outputs are unchanged, our measure of productivity will increase as bad outputs fall.

- I define a sustainable productivity index (SPI) to be any variable of the form:

$$SPI = \frac{GI^{1-\alpha} BI^{-\alpha}}{XI} \tag{1}$$

  where GI, BI and XI are indexes that measures changes in the volumes of good outputs, bad outputs and inputs respectively, and $\alpha \in [0, 1]$ measures the extent to which we want to account for bad outputs.

- I discuss alternative ways of constructing proper volume indexes and choosing the value of α. I use toy data to illustrate the properties of different indexes. I use national accounts data to compute SPI numbers for selected sectors of the Australian economy.

Ahead . . .

2. Proper Volume Indexes
3. Choosing the Value of α
4. Productivity Change in the Australian Economy
5. Concluding Remarks

2. Proper Volume Indexes

- Computing measures of output and input volume change involves assigning numbers to baskets of outputs and inputs.
- Measurement theory says that so-called index numbers must be assigned in such a way that the relationships between the numbers reflect the relationships between the baskets.
- O'Donnell (2016, 2018) defines a proper volume index to be one that satisfies a set of basic axioms from index theory (e.g., proportionality, transitivity). All proper indexes yield numbers that are consistent with measurement theory.

Example - Toy Data

Basket	Contents	Lowe	GY	Primal	BOD
A		1	1	1	1
M		2.032	1.779	2.996	2.048
R		2.032	1.779	2.996	2.048
W		2	2	2	2

Lowe uses a linear aggregator function and average prices as weights;
Geometric Young (GY) uses a double-log aggregator function and average value shares as weights;
Primal uses a linear aggregator function and representative shadow prices as weights;
Benefit-of-the-Doubt (BOD) uses uses a locally-linear aggregator function and variable weights.

Proper Volume Indexes (cont.)

Most economists and statistical agencies (including the OECD) use volume indexes that are not proper. They assign index numbers in ways that are not consistent with measurement theory

☹

Example - Toy Data

Basket	Contents	Fisher	CF	CT	CCD
A		1	1	1	1
M		1.892	2.389	1.942	2.088
R		1.893	2.854	1.943	2.089
W		2	3.642	2.027	1.791

The Fisher index fails a transitivity axiom. The chained Fisher (CF), chained Törnqvist (CT) and Caves-Christensen-Diewert (CCD) indexes all fail a proportionality axiom.

Why Aren't Statisticians Using Proper Indexes?

The main arguments:

1. Fisher (1922, pp. 274, 275) writes that "the only formulae [that satisfy transitivity] are index numbers that have constant weights . . . But, clearly, constant weighting is not theoretically correct". However, transitivity does not require constant weights (e.g., BOD indexes) and there is no credible theory that says weights must vary.

2. Landefeld et al (1995, p.31) write that "[the] use of fixed-weighted measures of real GDP . . . causes an overstatement of growth for periods after the base year and an understatement of growth for periods before the base year". However, while proper fixed-weight measures of volume change may overstate or understate growth in values (something they are not designed to measure), they will never overstate or understate growth in volumes.

Why Aren't Statisticians Using Proper Indexes?

3. Caves et al (1982) have a theorem saying that, under certain assumptions, Törnqvist indexes are equal to Malmquist indexes. However, (a) the assumptions behind the theorem cannot all be true, and (b) except in very restrictive cases, Malmquist indexes are not proper indexes, so who cares whether Tornqvist indexes are equal to Malmquist indexes?

4. Rao (2022, pp. 803, 805) argues that "the Lowe index . . . may lead to counter-intuitive conclusions". He considers a special case where the productivity gains from technical progress are exactly offset by the productivity losses from diseconomies of output substitution. Rao exclaims that "a shift in the frontier should imply productivity change driven by technical change. However, the Lowe index shows no productivity change!". However, Rao fails to distinguish the problem of measuring productivity change from the problem of explaining productivity change, and he fails to recognise that (dis)economies of substitution are important drivers of productivity change.

Why Aren't Statisticians Using Proper Indexes?

- Some statisticians may not be using proper indexes because they don't know enough about the properties of different indexes, they face bureaucratic inertia and/or they are incorrigible.
- Some economists and statisticians are pushing for change, e.g., the Shumway et al (2017) review of the USDA productivity accounts recommended that the USDA "replace the Caves-Christensen-Diewert spatial aggregator index with a Lowe, geometric Young or Färe-Primont index" (p.50).

3. Choosing the Value of α

1. Make a subjective/political choice.

2. Use the average estimated shadow value share of bad outputs. That is, estimate the shadow price of every output, then set α to the average of

$$\frac{\text{estimated shadow value of bad outputs}}{\text{estimated shadow value of all outputs}}$$

3. Use the value that minimises the amount of variation in the SPI numbers. This involves rewriting equation (1) as

$$\ln GI - \ln XI = \alpha[\ln GI + \ln BI] + e$$

where $e = \ln SPI$ is an unobserved variable that in most other contexts would be interpreted as statistical noise. If the design matrix is of full rank, then α can be estimated using least squares methods. If necessary, upper and lower bounds on the value of α can be incorporated into the estimation process.

Example - Toy Data

Using α = subjective choice = 0.1:

Row	g_1	g_2	b_1	b_2	x_1	x_2	ADEA	MDEA	BOD	LBOD	GBOD
A	1	1	1	1	1	1	1	1	1	1	1
B	1	1	1	1	0.56	0.56	1.786	1.786	1.786	1.786	1.786
C	2.37	2.37	1	1	1	1	2.174	2.174	2.174	2.174	2.174
D	2.11	2.11	0.4	0.4	1.05	0.7	2.765	2.588	2.584	2.750	2.583
E	1.81	3.62	0.3	0.4	1.05	0.7	3.658	3.299	3.289	3.483	3.120
:	:	:	:	:	:	:	:	:	:	:	:
V	1	5.166	1.8	1.5	1	1	2.767	2.237	2.722	2.697	2.084
W	2	2	0.5	0.5	0.919	0.919	2.176	2.176	2.176	2.176	2.176
X	1	1	3	2.7	1.464	0.215	1.842	1.875	2.579	1.803	1.871
Y	1	1	1	1	0.74	0.74	1.351	1.351	1.351	1.351	1.351
Z	1.81	3.62	0.3	0.4	2.1	1.4	1.829	1.650	1.644	1.742	1.560

ADEA = GI, BI and XI are all additive indexes with average DEA estimates of shadow prices as weights;
MDEA = GI, BI and XI are all multiplicative indexes with average DEA estimates of shadow value shares as weights;
BOD = GI, BI and XI are all BOD indexes;
LBOD = GI and XI are Lowe indexes and BI is a BOD index;
GBOD = GI and XI are GY indexes and BI is a BOD index.

As expected, the SPI numbers in any given column satisfy B $= 1/0.56 = 1.786$, C $= 2.37^{1-\alpha} = 2.174$, W $= 2/0.919 = 2.176$, Y $= 1/0.74 = 1.351$ and Z = 0.5E.

Example - Toy Data

Using DEA and α = average estimated shadow value share of bad outputs = 0.407:

Row	g_1	g_2	b_1	b_2	x_1	x_2	ADEA	MDEA	BOD	LBOD	GBOD
A	1	1	1	1	1	1	1	1	1	1	1
B	1	1	1	1	0.56	0.56	1.786	1.786	1.786	1.786	1.786
C	2.37	2.37	1	1	1	1	1.668	1.668	1.668	1.668	1.668
D	2.11	2.11	0.4	0.4	1.05	0.7	2.912	2.726	2.722	2.897	2.721
E	1.81	3.62	0.3	0.4	1.05	0.7	3.787	3.412	3.191	3.385	3.082
:	:	:	:	:	:	:	:	:	:	:	:
V	1	5.166	1.8	1.5	1	1	1.617	1.421	1.685	1.674	1.413
W	2	2	0.5	0.5	0.919	0.919	2.176	2.176	2.176	2.176	2.176
X	1	1	3	2.7	1.464	0.215	1.322	1.353	1.901	1.329	1.380
Y	1	1	1	1	0.74	0.74	1.351	1.351	1.351	1.351	1.351
Z	1.81	3.62	0.3	0.4	2.1	1.4	1.894	1.706	1.596	1.692	1.541

ADEA = GI, BI and XI are all additive indexes with average DEA estimates of shadow prices as weights;
MDEA = GI, BI and XI are all multiplicative indexes with average DEA estimates of shadow value shares as weights;
BOD = GI, BI and XI are all BOD indexes;
LBOD = GI and XI are Lowe indexes and BI is a BOD index;
GBOD = GI and XI are GY indexes and BI is a BOD index.

Again, as expected, the SPI numbers in any given column satisfy B = 1.786, C = $2.37^{1-\alpha}$ = 1.668, W = 2.176, Y = 1.351 and Z = 0.5E.

Example - Toy Data

Using the values of α that minimise the amount of variation in the SPI numbers:

Row	g_1	g_2	b_1	b_2	x_1	x_2	ADEA	MDEA	BOD	LBOD	GBOD
A	1	1	1	1	1	1	1	1	1	1	1
B	1	1	1	1	0.56	0.56	1.786	1.786	1.786	1.786	1.786
C	2.37	2.37	1	1	1	1	1.325	1.710	1.126	1.282	1.734
D	2.11	2.11	0.4	0.4	1.05	0.7	3.047	2.713	2.941	3.050	2.700
E	1.81	3.62	0.3	0.4	1.05	0.7	3.903	3.401	3.052	3.290	3.088
:	:	:	:	:	:	:	:	:	:	:	:
V	1	5.166	1.8	1.5	1	1	1.013	1.483	0.827	1.043	1.496
W	2	2	0.5	0.5	0.919	0.919	2.176	2.176	2.176	2.176	2.176
X	1	1	3	2.7	1.464	0.215	0.991	1.395	1.210	0.982	1.443
Y	1	1	1	1	0.74	0.74	1.351	1.351	1.351	1.351	1.351
Z	1.81	3.62	0.3	0.4	2.1	1.4	1.952	1.701	1.526	1.645	1.544
α							0.674	0.378	0.862	0.712	0.362

ADEA = GI, BI and XI are all additive indexes with average DEA estimates of shadow prices as weights;
MDEA = GI, BI and XI are all multiplicative indexes with average DEA estimates of shadow value shares as weights;
BOD = GI, BI and XI are all BOD indexes;
LBOD = GI and XI are Lowe indexes and BI is a BOD index;
GBOD = GI and XI are GY indexes and BI is a BOD index.

Again, as expected, the SPI numbers in any given column satisfy B = 1.786, C = $2.37^{1-\alpha}$, W = 2.176, Y = 1.351 and Z = 0.5E.

Other Indexes

Other proper SPIs include primal and dual indexes. Measures of productivity change that account for bad outputs but are not proper include:

- the measure of environmentally-adjusted MFP growth of Cardenas Rodriquez et al (2018): this measure is constructed in such a way that if all inputs and outputs are unchanged, then measured productivity growth is a random variable.
- the shadow CCD index of Pittman (1983).

Example - Toy Data

Row	g_1	g_2	b_1	b_2	x_1	x_2	Pittman (1983)
A	1	1	1	1	1	1	1
B	1	1	1	1	0.56	0.56	1.814
C	2.37	2.37	1	1	1	1	2.115
D	2.11	2.11	0.4	0.4	1.05	0.7	2.378
E	1.81	3.62	0.3	0.4	1.05	0.7	2.855
:	:	:	:	:	:	:	:
V	1	5.166	1.8	1.5	1	1	2.297
W	2	2	0.5	0.5	0.919	0.919	1.953
X	1	1	3	2.7	1.464	0.215	3.037
Y	1	1	1	1	0.74	0.74	1.404
Z	1.81	3.62	0.3	0.4	2.1	1.4	1.997

The P83 numbers do not satisfy B = 1.786, W = 2.176, Y = 1.351 or Z = 0.5E.

4. Productivity Change in the Australian Economy

National accounts data published by the Australian Bureau of Statistics (ABS). For each of 16 sectors of the market economy for the period YTJ 1995 to YTJ 2019:

- 1 good output (G = market output)
- 1 bad output (B = GHG emissions)
- 3 inputs (x_1 = capital, x_2 = labour, x_3 = other inputs)

Agriculture, Forestry and Fishing

YTJ	B0.1	L0.1	G0.1	BSS	LSS	GSS	BMV	LMV	GMV
1995	1	1	1	1	1	1	1	1	1
1996	1.070	1.056	1.057	1.065	1.051	1.052	1.014	1.018	1.019
1997	1.126	1.093	1.091	1.116	1.083	1.081	1.008	1.013	1.012
1998	1.154	1.108	1.106	1.148	1.102	1.100	1.082	1.061	1.059
1999	1.203	1.142	1.140	1.190	1.130	1.128	1.063	1.050	1.049
:	:	:	:	:	:	:	:	:	:
2015	1.342	1.300	1.298	1.332	1.291	1.289	1.237	1.230	1.229
2016	1.371	1.281	1.279	1.372	1.282	1.280	1.386	1.291	1.288
2017	1.453	1.322	1.320	1.453	1.322	1.320	1.454	1.322	1.320
2018	1.441	1.298	1.297	1.433	1.291	1.290	1.348	1.241	1.240
2019	1.417	1.282	1.281	1.427	1.291	1.290	1.536	1.354	1.353

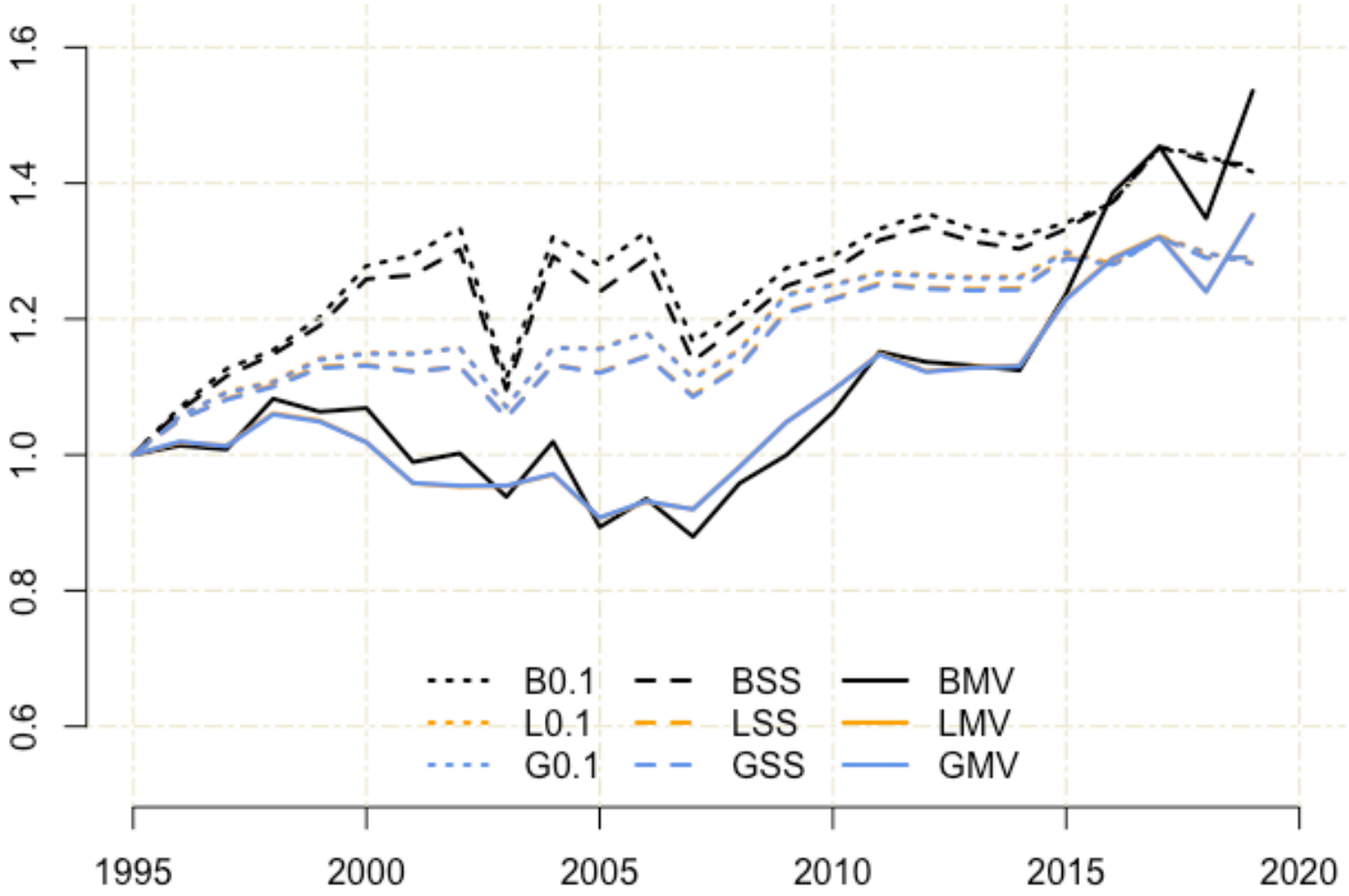
B0.1 = BOD input index and α = subjective choice = 0.1;
L0.1 = Lowe input index and α = subjective choice = 0.1;
G0.1 = GY input index and α = subjective choice = 0.1;

BSS = BOD input index and α = average estimated shadow value share of bad outputs = 0.148;
LSS = Lowe input index and α = average estimated shadow value share of bad outputs = 0.148;
GSS = GY input index and α = average estimated shadow value share of bad outputs = 0.148;

BMV = BOD input index and α = value that minimises the variation in the SPI numbers = 0.660;
LMV = Lowe input index and α = value that minimises the variation in the SPI numbers = 0.480;
GMV = GY input index and α = value that minimises the variation in the SPI numbers = 0.477;

Lowe almost identical to GY

Agriculture, Forestry and Fishing



Orange almost identical to blue

Retail Trade

YTJ	B0.1	L0.1	G0.1	BSS	LSS	GSS	BMV	LMV	GMV
1995	1	1	1	1	1	1	1	1	1
1996	0.989	1.004	1.001	0.973	0.988	0.984	0.991	1.006	1.002
1997	0.941	0.989	0.982	0.899	0.944	0.938	0.947	0.992	0.987
1998	0.930	0.989	0.980	0.865	0.920	0.912	0.939	0.994	0.987
1999	0.907	0.983	0.970	0.791	0.857	0.846	0.924	0.993	0.984
:	:	:	:	:	:	:	:	:	:
2015	0.917	0.955	0.948	0.521	0.542	0.538	0.991	0.998	1.007
2016	0.932	0.960	0.952	0.523	0.539	0.534	1.009	1.004	1.013
2017	0.943	0.956	0.947	0.519	0.526	0.521	1.023	1.001	1.009
2018	0.963	0.961	0.951	0.526	0.524	0.519	1.046	1.006	1.015
2019	0.969	0.946	0.936	0.508	0.496	0.491	1.058	0.994	1.003

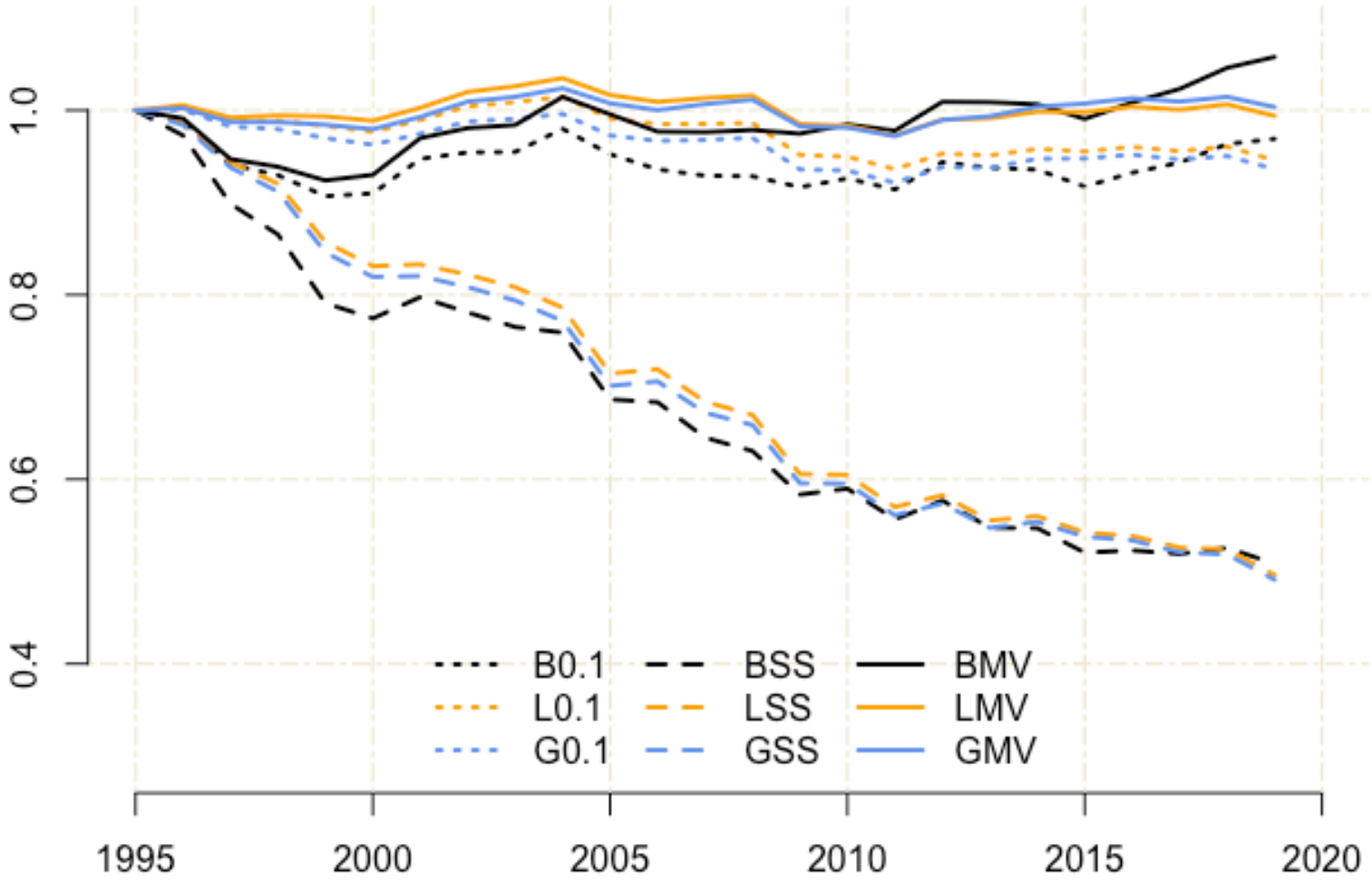
B0.1 = BOD input index and α = subjective choice = 0.1;
L0.1 = Lowe input index and α = subjective choice = 0.1;
G0.1 = GY input index and α = subjective choice = 0.1;

BSS = BOD input index and α = average estimated shadow value share of bad outputs = 0.49;
LSS = Lowe input index and α = average estimated shadow value share of bad outputs = 0.49;
GSS = GY input index and α = average estimated shadow value share of bad outputs = 0.49;

BMV = BOD input index and α = value that minimises the variation in the SPI numbers = 0.047;
LMV = Lowe input index and α = value that minimises the variation in the SPI numbers = 0.070;
GMV = GY input index and α = value that minimises the variation in the SPI numbers = 0.058;

Subjective choice of α relatively close to the α that minimises variation in SPI.

Retail Trade



Dotted relatively close to solid

5. Concluding Remarks

- Irrespective of the merits or otherwise of different indexes, when it comes to measuring volume change, the overriding consideration should be whether our indexes produce numbers that are consistent with measurement theory (i.e., whether the patterns in the numbers mirror the patterns in the volumes). Only proper volume indexes can do this.

- Proper indexes are generally easier to compute than the indexes used by most statistical agencies. Unlike those indexes, proper indexes can be used to make comparisons across both time and space (e.g., we can compare firm A in period 2 with firm C in period 5).

- We do not need any information about markets or production technologies (e.g., prices, value shares, shadow prices, marginal rates of transformation) in order to construct proper volume indexes. However, if that information is available, then we would probably want to use it..

- Weighted geometric averages of proper indexes are also proper $\Rightarrow$ we could construct a proper index by taking a geometric average of all the indexes discussed here (and others).

References I

Cardenas Rodriquez M, Hascic I, Souchier M (2018) Environmentally adjusted multifactor productivity: Methodology and empirical results for OECD and G20 countries. Ecological Economics 153:147–160

Caves D, Christensen L, Diewert W (1982) The economic theory of index numbers and the measurement of input, output, and productivity. Econometrica 50(6):1393–1414

Fisher I (1922) The Making of Index Numbers. Houghton Mifflin, Boston

Landefeld J, Parker R, Triplett J (1995) Preview of the comprehensive revision of the national income and product accounts: BEA's new featured measures of output and prices. Survey of Current Business 75(July):31–38

O'Donnell C (2016) Using information about technologies, markets and firm behaviour to decompose a proper productivity index. Journal of Econometrics 190(2):328–340

O'Donnell C (2018) Productivity and Efficiency Analysis: An Economic Approach to Measuring and Explaining Managerial Performance. Springer Nature, Singapore

OECD (2001) OECD Manual: Measuring Productivity: Measurement of Aggregate and Industry-Level Productivity Growth. Organisation for Economic Development and Cooperation, Paris

Pittman R (1983) Multilateral productivity comparisons with undesirable outputs. The Economic Journal 93(372):883–891

Rao D (2022) Index numbers and productivity measurement. In: Ray S, Chambers R, Kumbhakar S (eds) Handbook of Production Economics, Springer Nature, Singapore, chap 19

References II

Shumway CR, Fraumeni BM, Fulginiti LE, Samuels JD, Stefanou SE (2017) Measurement of U.S. agricultural productivity: A 2014 review of current statistics and proposals for change. Contractor and Cooperator Report CCR-69, U.S. Department of Agriculture